Skin effect & Skin depth,

Penetration of high frequency EM-fields in to a good conductor:

Skin effect:

Skin effect is the tendency of an alternating current to become distributed with in a conductor such that current density is largest near the surface of a conductor at the highest frequency and decreases with greater depth in the conductor.

or The gradual shift of the alternating current from the centre to the surface of a conductor due to the increase in frequency is known as Skin effect.

Skin depth: The skin depth is a measure of depth of a conductor at which the current density falls to "1/e" of its value near the surface ie 37% of the peak value;

Penetration of high frequency EM-fields in to a good Conductor:

Let us consider Those Conductors who obey ohm's Law

$$\bar{J} = \sigma_c \bar{E} \quad \text{--- (1)}$$

Where σ_c is the Conductivity

We have Maxwell's equations

$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

Let $D = D_0 e^{j\omega t}$

$E = E_0 e^{j\omega t}$

& $H = H_0 e^{j\omega t}$

$$\frac{\partial \bar{D}}{\partial t} = j\omega \bar{D}$$

$$\bar{\nabla} \times \bar{H} = \bar{J} + j\omega \bar{D}$$

$$\bar{\nabla} \times \bar{H} = (\sigma + j\omega\epsilon)\bar{E} \quad \text{--- (2)}$$

In the absence of charges $\rho = 0$

$$\bar{\nabla} \cdot \bar{E} = 0 \quad \Rightarrow \bar{\nabla} \cdot \bar{D} = \rho = 0$$

We assume that there exists the homogeniety of σ & ϵ.

To derive a differential equation which determines the penetration of the fields in to the Conductor we take $\bar{\nabla} \times \bar{E} = -\frac{\partial B}{\partial t}$ $\therefore B = \mu H$

$$\bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H} \quad \text{--- (3)}$$

Taking curl of both Sides of eq 3,

Contd:

$$\bar{\nabla}\times\bar{\nabla}\times\bar{E} = -j\omega\mu(\bar{\nabla}\times\bar{H}) \quad (4)$$

Applying vector I.D to the L.H.S

$$\bar{\nabla}\times\bar{\nabla}\times\bar{E} = \bar{\nabla}(\cancel{\bar{\nabla}\cdot\bar{E}}) - \nabla^2\bar{E}$$

$$\bar{\nabla}\cdot\bar{E} = 0 \quad \text{as } \rho = 0$$

$$\cancel{-}\nabla^2\bar{E} = \cancel{-}j\omega\mu\sigma_c\bar{E}$$

$$\nabla^2\bar{E} = j\omega\mu\sigma E \quad (5) \Rightarrow$$

Assuming
[Considering $(\sigma + j\omega\epsilon)$
$\sigma >>> \epsilon\omega$]

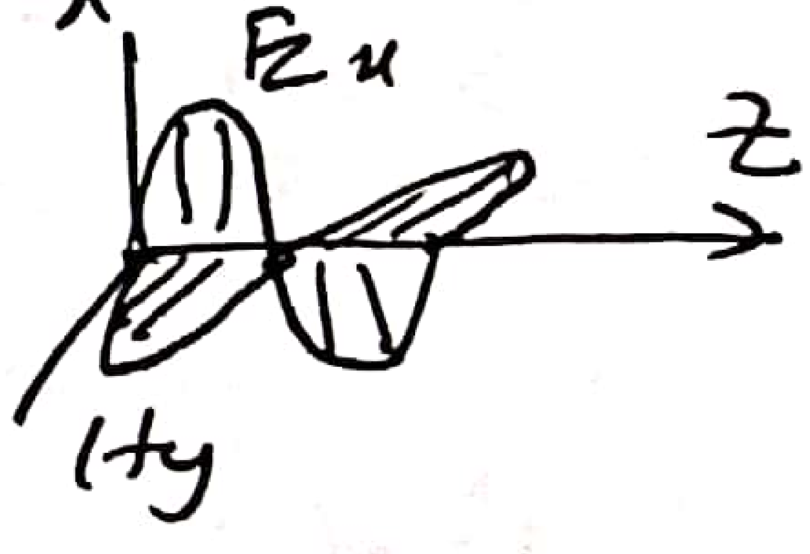
Similarly $\nabla^2 H = j\omega\mu\sigma H \quad (6)$

put $J = \sigma E$ or $E = J/\sigma$ in eq (5)

$$\nabla^2\bar{J} = j\omega\mu\bar{J} \quad (7)$$

Hint
$\bar{\nabla}\times\bar{\nabla}\times\bar{H} = \bar{\nabla}\times J$
$J = \sigma\bar{E}$
$\bar{\nabla}\times\bar{E} = -\frac{\partial B}{\partial t}$
$\nabla^2 H = \bar{\nabla}\times\bar{J}$

Let us consider a uniform field situation with Electric field vector moving along the Z direction.



$$\frac{d^2E_z}{dx^2} = j\omega\mu\sigma E_z$$

$$= \tau^2 E_z \quad (8)$$

Where $\tau^2 = j\omega\mu\sigma \quad (9)$

We can write $j = \left(\frac{1+j}{\sqrt{2}}\right)^2$

or $\sqrt{j} = \left(\frac{1+j}{\sqrt{2}}\right)$

$$\tau = \sqrt{j\omega\mu\sigma} = \sqrt{j}\sqrt{\omega\mu\sigma}$$

Contd.

$$\gamma = \left(\frac{1+j}{\sqrt{2}}\right)\sqrt{\omega\mu\sigma} \quad \text{—(10)}$$

$\omega = 2\pi f$

$$= \left(\frac{1+j}{\sqrt{2}}\right)\sqrt{2\pi f\mu\sigma}$$

$$= (1+j)\sqrt{\pi f\mu\sigma}$$

$$= \frac{1+j}{\delta}, \quad \text{where } \delta = \left(\frac{1}{\sqrt{\pi f\mu\sigma}}\right)$$

& δ is called the Skin depth

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} \; (m) \quad \text{—(11)}$$

A complete Solution of eq (8) is of the form

$$E_z = C_1 e^{-\gamma x} + C_2 e^{+\gamma x} \quad \text{—(12)}$$

at $x = \infty$ the E_z field will increase to the impossible value of ∞ ($e^{+\infty\gamma}$) so unless C_2 is zero. $C_2 = 0$

& Coefficient $C_1 = E_0$ is Amplitude of the field.

$E_z = E_0$ at $x = 0$, $C_1 = E_0$

$$E_z = E_0 \, e^{-\gamma x}$$

$$E_z = E_0 \, e^{-\left(\frac{1+j}{\delta}\right)x} \quad \text{—(13)}$$

$$E_z = E_0 \, e^{-x/\delta} \times e^{-jx/\delta} \quad \text{—(14)}$$

Contd

Similarly

$$H_y = H_0 \, e^{-x/\delta} \cdot e^{-jx/\delta} \quad \text{—(15)}$$

&

$$J_z = J_0 \, e^{-x/\delta} \cdot e^{-jx/\delta} \quad \text{—(16)}$$

The magnetic field & current density are governed by the same differential equations as the E field.

Conclusion: It is evident from equation (14) to eqn (16) that the magnitudes of the fields & current decreases exponentially with penetration in to the conductor & δ has the significance of the depth at which they have decreased to $\left(\frac{1}{e}\right)$ about (36.9%) of their peak values at the surface. The quantity δ is called the depth of penetration or Skin depth.

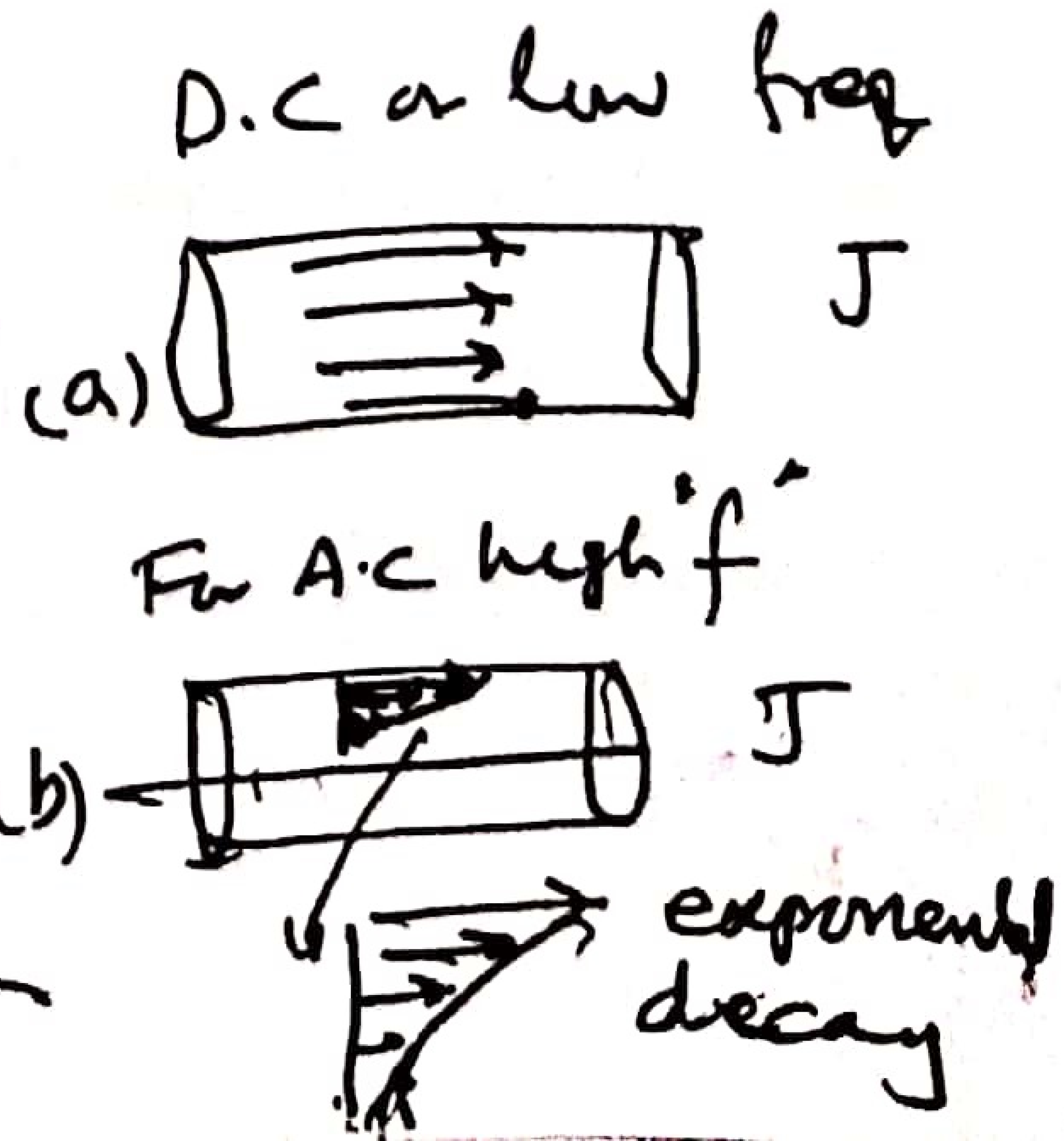
$$|E_z| = E_0 e^{-x/\delta} \quad \text{at } x = \delta$$

$$E_z = E_0 e^{-1} = \frac{E_0}{e} = 0.369 \, E_0$$

$$E_z = 37\% \, E_0$$

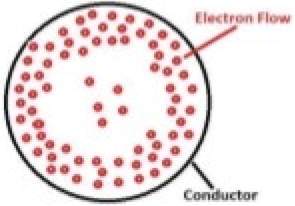
f = 10 GHz

Conductor	Skin depth (μm)	At 50 Hz, δ μm
Al	0.8 μm	
Cu	0.65 μm	9360 μm
Gold	0.79 μm	
Silver	0.64 μm	



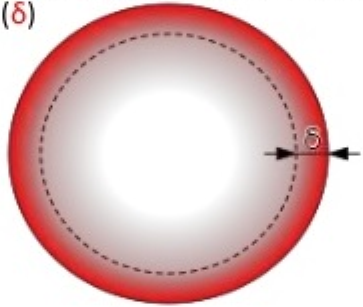
What is Skin Effect?

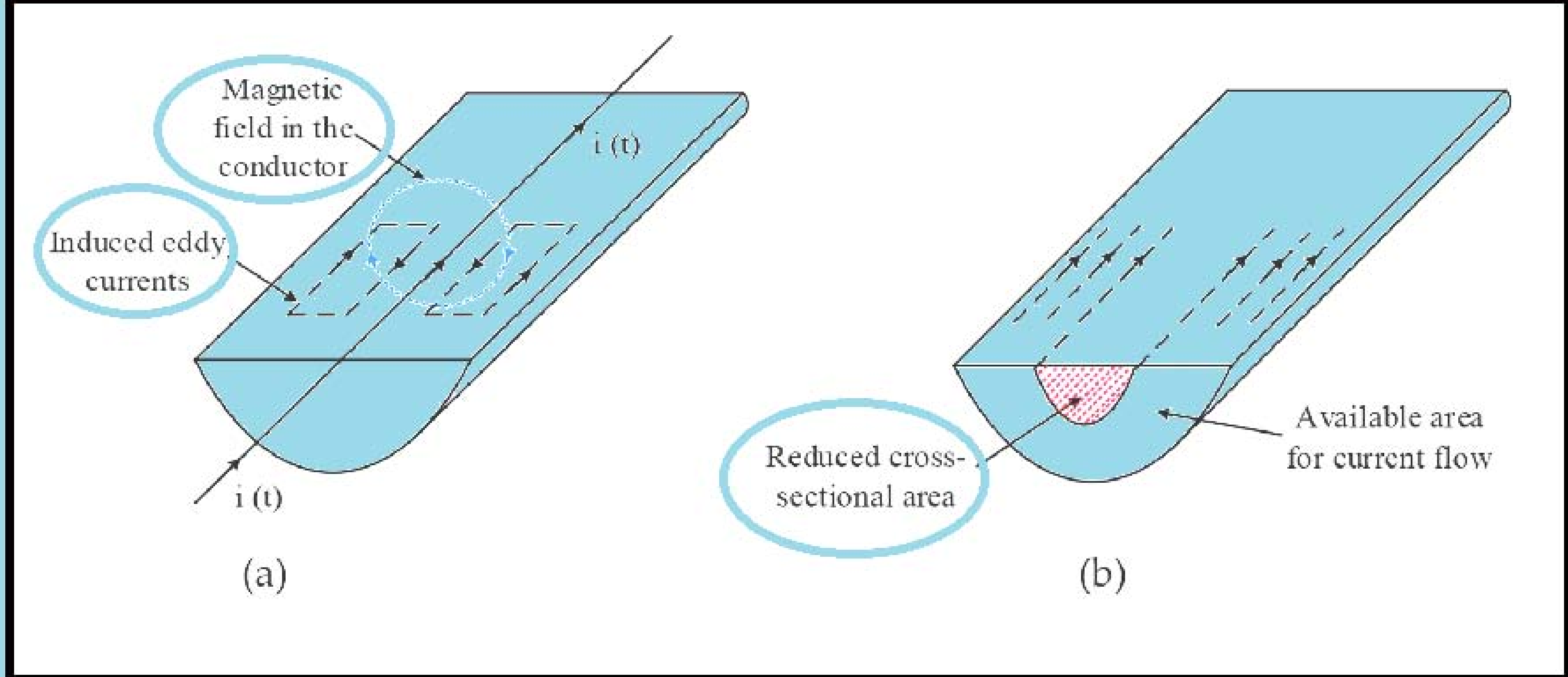
- **Skin effect** is the tendency of an alternating electric current (AC) to become distributed within a conductor such that the current density is largest near the surface of the conductor, and decreases with greater depths in the conductor.



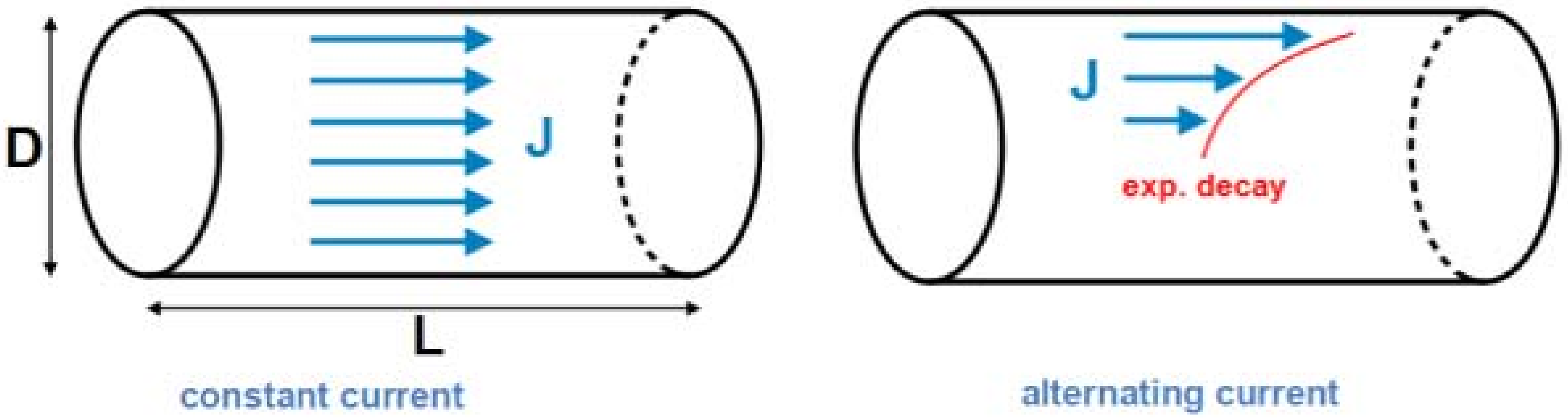
Skin Depth(δ)

- *Skin depth* is a measure of the depth at which the current density falls to 1/e of its value near the surface
- Skin depth also describes the exponential decay of the electric and magnetic fields, as well as the density of induced currents
- Distribution of current flow in a cylindrical conductor, For alternating current, most (63%) of the electric current flows between the surface and the skin depth (δ)

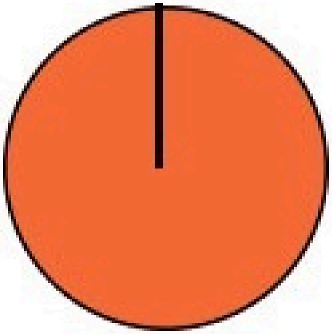



Magnetic field in the conductor
i (t)
Induced eddy currents
i (t)
(a)
Reduced cross-sectional area
Available area for current flow
(b)

Skin Depth(δ)

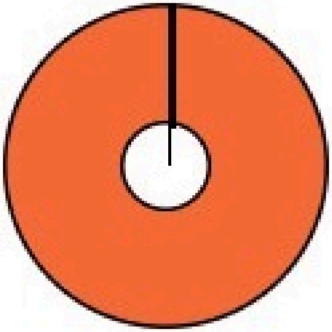


Copper Wire, Radius=10 mm



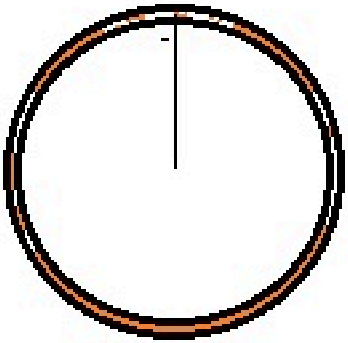
Cross-sectional area of a round conductor available for conducting DC current

No Skin depth phenomenon, all area of crossection available for current.



Cross-sectional area of the same conductor available for conducting low-frequency AC

f=50Hz, skin depth= 9.3mm



Cross-sectional area of the same conductor available for conducting high-frequency AC

f=10GHz, skin depth=0.00065 mm
=0.65μm